MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2016/2017

TMA1211 – DISCRETE MATHEMATICS AND PROBABILITY

(All sections / Groups)

27 FEBRUARY 2017
2.30 p.m. – 4.30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This Question paper consists of **8 pages** only excluding the cover page with **5 Questions** and an **Appendix**.
2. Attempt **ALL FIVE(5)** questions.
3. Please write your answers in the Answer Booklet provided, and **start each question on a new page.**
4. Show all steps.
5. Statistical tables and formulas are provided and attached in Appendix.

Answer ALL FIVE (5) questions.

QUESTION 1 (10 marks)

a) By using **equivalence laws** (not using truth table), prove that $q \wedge (p \rightarrow \neg q)$ is logically equivalent to $q \wedge \neg p$, i.e. $q \wedge (p \rightarrow \neg q) \Leftrightarrow q \wedge \neg p$. [2 marks]

b) Given $R = \{(1,2), (2,1), (2,2), (3,1)\}$ and $S = \{(1,1), (2,3), (3,2), (4,1)\}$.
   i. Represent R with a directed graph (diagraph). [1 mark]
   ii. Find $S \circ R$. [1 mark]
   iii. Find R^{-1} . [1 mark]

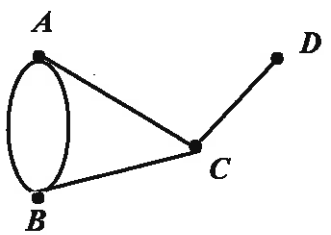
c) Given the functions f and g where $f : R \rightarrow R, f(x) = x^3 + 2x^2 + 1$ and $g : R \rightarrow R$, $g(x) = 5 - x$. Find
   i. $f + g$. [1 mark]
   ii. $g \circ g$. [2 marks]
   iii. g^{-1}. [2 marks]

QUESTION 2 (10 marks)

a) Determine whether $Q = \{3,5\}$ is **closed** under the operation of
   i. addition. [2 marks]
   ii. multiplication. [2 marks]

b) Consider the set R of real numbers and let * be the operation on R defined by $a * b = 2ab + b$.
   Is $(R,*)$ a semigroup? [4 marks]

c) Given a graph as below.



   i. Find the degree of each vertex. [1 mark]
   ii. Does the graph above has an Euler circuit? [1 mark]

Continued.....

QUESTION 3 (10 marks)

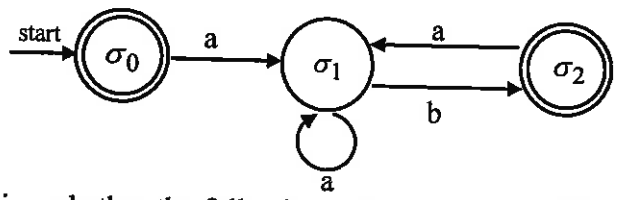
a) If $f(n)$ is defined recursively by $f(0) = -1$, $f(1) = 2$ and $f(n+1) = f(n) + 2f(n-1)$ for $n = 1, 2, ...$
Find $f(2)$ and $f(3)$.
[3 marks]

b) Given the following state table with the finite-state machine $(S, I, O, f, g, \sigma_0)$ where $S = \{\sigma_0, \sigma_1, \sigma_2\}$, $I = \{a, b\}$ and $O = \{0,1\}$.

	f		g	
S \ I	a	b	a	b
σ_0	σ_0	σ_1	0	1
σ_1	σ_1	σ_2	1	1
σ_2	σ_1	σ_2	1	0

i. Draw the transition diagram of the finite-state machine. [3 marks]
ii. Then, based on the transition diagram or other method, find the output strings of the following input strings.
- *aababb* [1 mark]
- *abaaba* [1 mark]

c) Given below is the state diagram for the finite-state automaton (S, I, f, σ_0, F) where $S = \{\sigma_0, \sigma_1, \sigma_2\}$, $I = \{a, b\}$ and $F = \{\sigma_0, \sigma_2\}$.



Determine whether the following strings are accepted by the Deterministic Finite Automata (DFA) or not.
i. *abaab* [1 mark]
ii. *aababa* [1 mark]

Continued.....

QUESTION 4 (10 marks)

a) Alyssa is setting a 4-digit passcode for her new smartphone. The passcode is to formed by using digits 0 to 9 at which the digit can be repeated.

i. How many different passcode can be formed? [2 marks]

ii. How many different passcode can be formed if she wants to use '2' as the first digit of the passcode? [1 mark]

iii. Using your answers in a)i and a)ii or other method, find the probability of having the passcode as she wants in a)ii above? [1 mark]

b) A random sample of 1500 IT executives is selected. They are classified by gender and type of employment (full time or part time) as in the table below.

	Full Time (FT)	Part Time (PT)	Total
Male (M)	380	255	635
Female (F)	620	245	865
Total	1000	500	1500

If an IT executive is selected at random. Let M = {a male is selected}, F = {a female is selected}, FT = {a full time executive is selected}, and PT = {a part time executive is selected}. Find

i. $P(F)$. [2 marks]

ii. $P(F \cup PT)$. [2 marks]

iii. $P(F|PT)$. [2 marks]

Continued.....

Question 5 (10 marks)

a) On average, there are 2 system errors occur in a week. By using statistical table, find the probability of no error occurs on any given week. [1 mark]

b) Time taken by all participants to complete the task in Hacker Competition is normally distributed with a mean of 2 hours and a standard deviation of 0.5 hour. If a participant is randomly selected, what is the probability that he will complete the task in less than 2.5 hours? [2 marks]

c) The random variable X has a range of $\{1, 2, 3\}$ and the random variable Y has a range of $\{0, 1, 2\}$. Their joint probability distribution is given as follows:

$P(x,y)$		y		
		0	1	2
x	1	k	k	$2k$
	2	0.04	$2k$	$6k$
	3	0.01	$4k$	$3k$

i. Find the value of k. [2 marks]

If $k = 0.05$, find

ii. $P(X = Y)$. [1 mark]

iii. the marginal probability distribution of X. [2 marks]

d) Given the joint probability density function of X and Y is as below.

$$f(x,y) = \begin{cases} 4xy & , \quad 0 < x < 1,\ 0 < y < 1 \\ 0 & , \quad \textit{otherwise} \end{cases}$$

Find the marginal probability distribution of Y. [2 marks]

Continued.....

Appendix

Table: Important Equivalences

Identity laws	Negation laws	Others
$p \wedge T \Leftrightarrow p$	$p \vee \neg p \Leftrightarrow T$	$p \rightarrow q \Leftrightarrow \neg p \vee q$ (CI)
$p \vee F \Leftrightarrow p$	$p \wedge \neg p \Leftrightarrow F$	
Domination laws	**Absorption Laws**	
$p \vee T \Leftrightarrow T$	$p \vee (p \wedge q) \Leftrightarrow p$	
$p \wedge F \Leftrightarrow F$	$p \wedge (p \vee q) \Leftrightarrow p$	
Idempotent laws	**De Morgan's laws**	
$p \vee p \Leftrightarrow p$	$\neg(p \wedge q) \Leftrightarrow (\neg p) \vee (\neg q)$	
$p \wedge p \Leftrightarrow p$	$\neg(p \vee q) \Leftrightarrow (\neg p) \wedge (\neg q)$	
Double Negation laws	**Commutative laws**	
$\neg(\neg p) \Leftrightarrow p$	$p \vee q \Leftrightarrow q \vee p$	
	$p \wedge q \Leftrightarrow q \wedge p$	
Associative laws	**Distributive laws**	
$(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$	$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$	
$(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$	$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$	

Formulas

Probability

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

Standardize to *z*-value

- $z = \dfrac{X - \mu}{\sigma}$

Joint Discrete Probability Distribution

- $\sum_x \sum_y P(x, y) = 1$

Joint Continuous Probability Distribution

- Marginal probability distribution of X, $f(x) = \int_y f(x, y)\, dy$

Continued.....

Statistical Table 1: Cumulative Poisson Probabilities

Table A.2 Cumulative Poisson Probabilities *(cont.)*

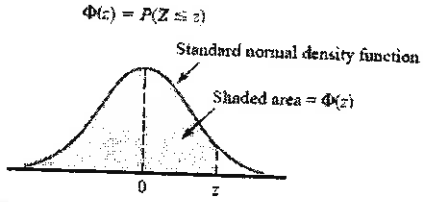
$$F(x;\lambda) = \sum_{y=0}^{x} \frac{e^{-\lambda}\lambda^{y}}{y!}$$

	λ										
x	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	15.0	20.0
0	.135	.050	.018	.007	.002	.001	.000	.000	.000	.000	.000
1	.406	.199	.092	.040	.017	.007	.003	.001	.000	.000	.000
2	.677	.423	.238	.125	.062	.030	.014	.006	.003	.000	.000
3	.857	.647	.433	.265	.151	.082	.042	.021	.010	.000	.000
4	.947	.815	.629	.440	.285	.173	.100	.055	.029	.001	.000
5	.983	.916	.785	.616	.446	.301	.191	.116	.067	.003	.000
6	.995	.966	.889	.762	.606	.450	.313	.207	.130	.008	.000
7	.999	.988	.949	.867	.744	.599	.453	.324	.220	.018	.001
8	1.000	.996	.979	.932	.847	.729	.593	.456	.333	.037	.002
9		.999	.992	.968	.916	.830	.717	.587	.458	.070	.005
10		1.000	.997	.986	.957	.901	.816	.706	.583	.118	.011
11			.999	.995	.980	.947	.888	.803	.697	.185	.021
12			1.000	.998	.991	.973	.936	.876	.792	.268	.039
13				.999	.996	.987	.966	.926	.864	.363	.066
14				1.000	.999	.994	.983	.959	.917	.466	.105
15					.999	.998	.992	.978	.951	.568	.157
16					1.000	.999	.996	.989	.973	.664	.221
17						1.000	.998	.995	.986	.749	.297
18							.999	.998	.993	.819	.381
19							1.000	.999	.997	.875	.470
20								1.000	.998	.917	.559

Continued.....

Statistical Table 2: Standard Normal Distribution

Table A.3 Standard Normal Curve Areas



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0038
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3482
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Continued.....

Statistical Table 2: Standard Normal Distribution (Continued)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

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